Supervised Learning-Based Ideal Observer Approximation for Joint Detection and Estimation Tasks

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CHALLENGES

- Optimization of medical imaging system performance should be guided by task-based measures of image quality (IQ), which quantifies the ability of an observer to perform specific tasks.
- The estimation ROC curve (EROC) [1] has been proposed as a figure-of-merit (FOM) for evaluating the performance of an observer on general joint detection and estimation tasks. The ideal EROC observer set up the upper limit of all other observers.
- Approximating the ideal EROC observer is a more challenging problem compared to approximating IO for single detection task or joint detection/localization task, mainly because the EROC observer test statistic [1] is defined as an integral function of the ideal likelihood ratio and the ideal estimate, which depends on the estimate of parameters.
- It is difficult to design a single loss function for approximating the EROC-IO for joint detection and estimation tasks by use of supervised deep-learning methods.

INNOVATION

- The EROC-IO test statistic is innovatively decomposed into a multiplication of ideal likelihood ratio and utility weighted posterior mean. A multi-task convolutional neural network (CNN) is constructed for approximating the ideal estimate and the ideal likelihood ratio, and ultimately for EROC-IO approximation.
- The proposed method is an alternative approach to conventional numerical approaches and can approximate the ideal EROC observer test statistic and estimate in complex cases.

IDEAL EROC OBSERVER

- For a given joint detection/estimation task, the IO test statistic $T_I(\mathbf{g})$ can be represented as:

$$T_I(\mathbf{g}) = \int pr(\boldsymbol{\theta})\Lambda(\mathbf{g}|\boldsymbol{\theta})u\big(\widehat{\boldsymbol{\theta}}_{\boldsymbol{I}}(\mathbf{g}),\boldsymbol{\theta}\big)d\boldsymbol{\theta}.$$

- Here $\Lambda(\mathbf{g}|\boldsymbol{\theta}) = \frac{pr(\mathbf{g}|\boldsymbol{\theta},H_1)}{pr(\mathbf{g}|H_0)}$ is a conditional likelihood ratio, $\boldsymbol{\theta}$ represents the true parameter vector associated with the signal, and $u\big(\widehat{\boldsymbol{\theta}}_{\boldsymbol{I}}(\mathbf{g}),\boldsymbol{\theta}\big)$ is the utility function of the estimate $\widehat{\boldsymbol{\theta}}_{\boldsymbol{I}}(\mathbf{g})$ when the signal is actually present.
- The IO estimate $\widehat{\boldsymbol{\theta}}_{\boldsymbol{I}}(\mathbf{g})$ can be represented as:

$$\widehat{\boldsymbol{\theta}}_{\boldsymbol{I}}(\mathbf{g}) = \underset{\boldsymbol{\theta}'}{\operatorname{argmax}}\left\{\int pr(\boldsymbol{\theta})\Lambda(\mathbf{g}|\boldsymbol{\theta})u\big(\widehat{\boldsymbol{\theta}}_{\boldsymbol{I}}(\mathbf{g}),\boldsymbol{\theta}\big)d\boldsymbol{\theta}\right\}.$$

- The utility weighted posterior mean $U(\mathbf{g})$ can be represented as:

$$U(\mathbf{g}) = \int pr(\boldsymbol{\theta}|\mathbf{g},H_1)u\big(\widehat{\boldsymbol{\theta}}_{\boldsymbol{I}}(\mathbf{g}),\boldsymbol{\theta}\big)d\boldsymbol{\theta}$$

- Without loss of generality, $T_I(\mathbf{g})$ can be decomposed into the multiplication of $\Lambda(\mathbf{g})$ and $U(\mathbf{g})$ by using Bayes' rule:

$$T_I(\mathbf{g}) = \int \frac{pr(\mathbf{g}|H_1)}{pr(\mathbf{g}|H_0)}pr(\boldsymbol{\theta}|\mathbf{g},H_1)u\big(\widehat{\boldsymbol{\theta}}_{\boldsymbol{I}}(\mathbf{g}),\boldsymbol{\theta}\big)d\boldsymbol{\theta} = \Lambda(\mathbf{g})U(\mathbf{g}).$$

SUPERVISED-LEARNING BASED APPROACH

- We proposed to approximate ideal EROC observer with joint supervised learning and Markov-Chain Mote Carlo (MCMC) strategy.
- A multi-task convolutional neural network (CNN) is constructed for approximating the test statistic of EROC-IO for a joint detection and estimation task.

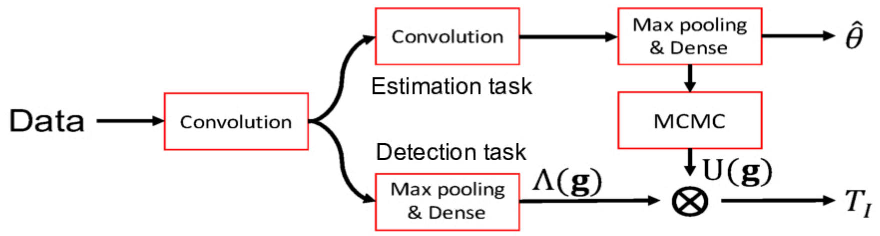

Fig. 1. The multi-task CNN architecture employed for approximating the ideal EROC observer test statistic. The first part of convolution layers are shared for both tasks.

- As shown in **Fig. 1**, the CNN for the detection task is to approximate the ideal likelihood ratio $\Lambda(\mathbf{g})$, and the CNN for the estimation task is to directly approximate the ideal estimate $\widehat{\boldsymbol{\theta}}_{\boldsymbol{I}}(\mathbf{g})$ for signal-present hypothesis.
- The utility weighted posterior mean U can be estimated by applying Monte Carlo integration on the approximated $\widehat{\boldsymbol{\theta}}_{\boldsymbol{I}}(\mathbf{g})$. The estimated $\widehat{U}(\mathbf{g}) = \frac{1}{J}\sum_{j=1}^{J} u(\widehat{\boldsymbol{\theta}}_{\boldsymbol{I}}(\mathbf{g}),\boldsymbol{\theta}^{(j)})$, where $\boldsymbol{\theta}^{(j)}$ is a sample from the density $pr(\boldsymbol{\theta}|\mathbf{g},H_1)$.
- A Markov chain can be constructed with $\alpha = \min\left[1,\frac{pr(\mathbf{g}|\widetilde{\boldsymbol{\theta}},H_1)pr(\widetilde{\boldsymbol{\theta}})q(\widetilde{\boldsymbol{\theta}}|\boldsymbol{\theta}^{(j)})}{pr(\mathbf{g}|\boldsymbol{\theta}^{(j)},H_1)pr(\boldsymbol{\theta}^{(j)})q(\boldsymbol{\theta}^{(j)}|\widetilde{\boldsymbol{\theta}})}\right]$, where α is the acceptance rate, $\widetilde{\boldsymbol{\theta}}$ is the candidate vector when given $\boldsymbol{\theta}^{(j)}$, and $q(\widetilde{\boldsymbol{\theta}}|\boldsymbol{\theta}^{(j)})$ is the proposal density which is designed to be symmetric, i.e. $q(\widetilde{\boldsymbol{\theta}}|\boldsymbol{\theta}^{(j)}) = q(\boldsymbol{\theta}^{(j)}|\widetilde{\boldsymbol{\theta}})$.
- The ideal test statistic $T_I(\mathbf{g})$ can be approximated by multiplying $\widehat{U}(\mathbf{g})$ and $\Lambda(\mathbf{g})$.
- The loss function of the estimation task for approximate $\widehat{\boldsymbol{\theta}}_{\boldsymbol{I}}(\mathbf{g})$ is defined as the negative of the utility function because this definition also minimizes the Bayesian risk [1].
- The loss function of the detection task for approximate $\Lambda(\mathbf{g})$ is defined as sigmoid cross entropy considering it is a monotonic transformation of the likelihood ratio [2].
- The multi-task CNN optimizes the two loss functions sequentially in one iteration.

EXPERIMENTS

- The ability of the proposed method to approximate ideal EROC observer is explored under the background−known−exactly (BKE) and background−known−statistically (BKS) cases. The joint detection and estimation task defined in this study is to detect a known signal with unknown amplitude with Gaussian noise.
- The imaging system is defined as $\mathbf{g} = \mathcal{H}f(\mathbf{r}) + \mathbf{n}$, where g is measurement data, $\mathcal{H}$ is a continuous-to-discrete imaging operator, $f(\mathbf{r})$ is the object function with a spatial coordinate $\mathbf{r}$ and $\mathbf{n}$ is measurement noise.
- In BKE case, the background is defined as zero. In BKS case, type 1 lumpy object model and type 2 lumpy object model (correlated Gaussian background) were considered. The joint tasks can be viewed as a surrogate for tumor detection in positron emission tomography (PET) images, where the task is to detect a lesion and estimate its maximum standardized uptake value [3].

RESULTS

- In BKE case and the BKS case with type 2 lumpy background, the IO test statistic and estimate can be analytically determined, which will be compared to demonstrate the approximation based on the proposed method.

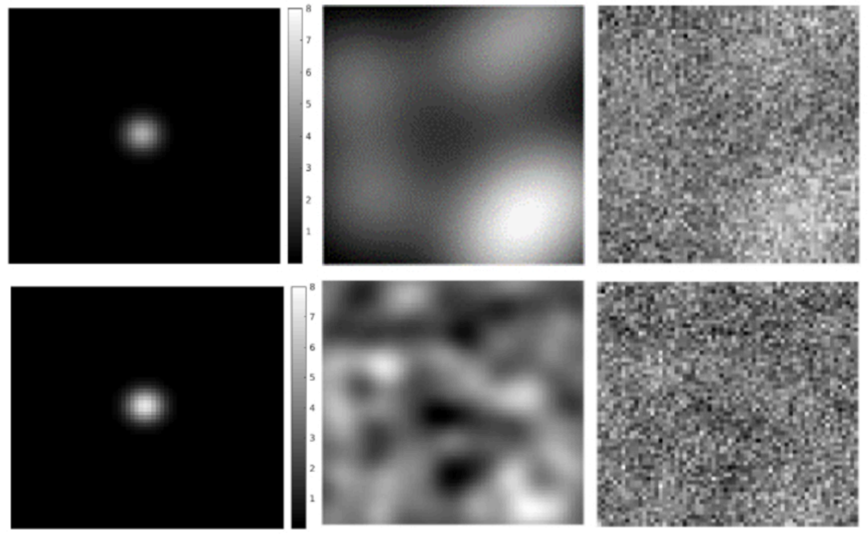

Fig. 2. Top and bottom rows show images with type 1 and type 2 lumpy background, respectively. These images are employed for the BKS study. Images shown from left to right on each row are examples of signals (left), signal-present measurement without noise (middle), signal-present noise measurement (right).

- In BKS case with type 1 lumpy background, the approximated IO is compared with a numerical joint detection and estimation observer called channelized joint observer (CJO) [4] because the ideal estimate is intractable for conventional methods such as MCMC for a Gaussian utility function.

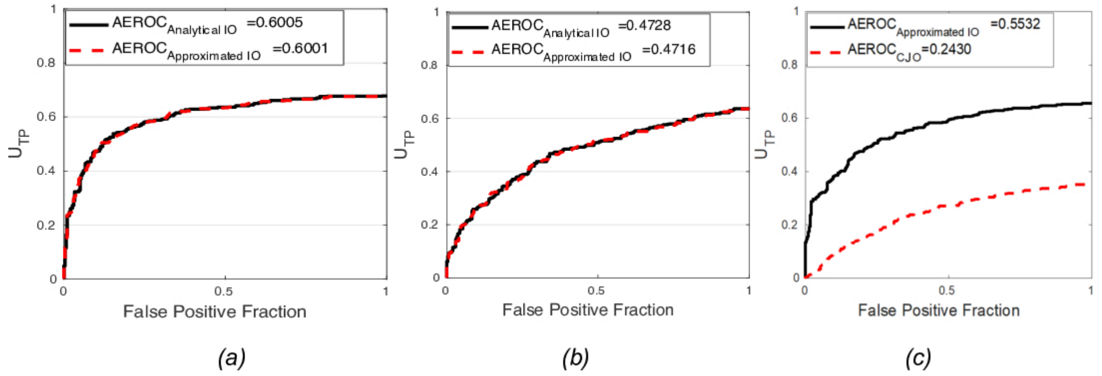

Fig. 3. Testing EROC curves for the IO approximations in different cases. (a) BKE case; (b) Type 2 lumpy background for BKS case; (c) Type 1 lumpy background for BKS case.

- For BKE case and BKS case with type 2 lumpy background models (Fig.3 (a) &(b)), the resulted AEROC values were close to those of the analytical IO, respectively.
- For BKS case with type 1 lumpy background model (Fig. 3(c)), the resulting AEROC value was much greater than that obtained by CJO. Because CJO is a numerical observer and its EROC curve should lie under that produced by IO, Fig. 3(c) indicates that the performance of the approximated IO with the proposed method is much better than CJO.

MULTI-TASK CNN TRAINING PERFORMANCE

- As shown in Fig.4, the loss functions for the detection and estimation tasks can converge although the network minimizes the two loss functions sequentially in one iteration.
- The convolution layers for the detection task and estimation task share some common features, and the noise adding strategy is effective to prevent overfitting.

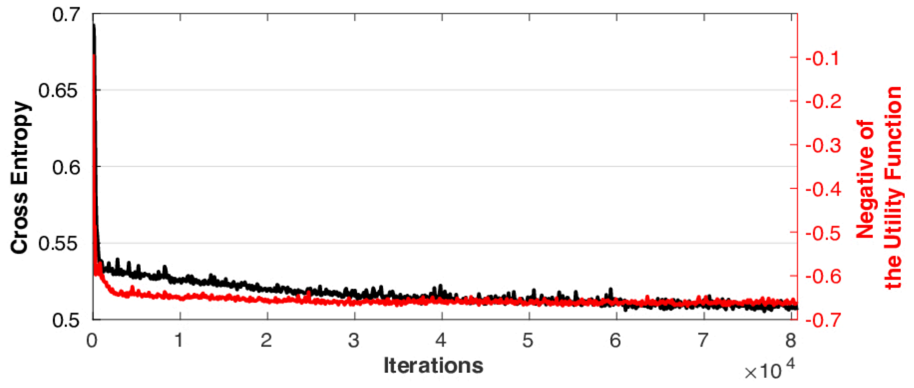

Fig. 4. Validation loss with respect to the iterations on BKS case with type 1 lumpy background.

REFERENCES

[1] E. Clarkson, "Estimation receiver operating characteristic curve and ideal observers for combined detection/estimation tasks," J. Opt. Soc. Am. A (2007)

[2] W. Zhou, H. Li and M. A. Anastasio, "Approximating the Ideal Observer and Hotelling Observer for Binary Signal Detection Tasks by Use of Supervised Learning Methods," in IEEE Transactions on Medical Imaging, vol. 38, no. 10, pp. 2456-2468, Oct. 2019.

[3] A. Wunderlich, B. Goossens and C. K. Abbey, "Optimal Joint Detection and Estimation That Maximizes ROC-Type Curves," in IEEE Transactions on Medical Imaging, vol. 35, no. 9, pp. 2164-2173, Sept. 2016.

[4] L. Zhang, B. Goossens, C. Cavaro-Ménard, Patr Le Callet, and Di Ge, "Channelized model observer for the detection and estimation of signals with unknown amplitude, orientation, and size," J. Opt. Soc. Am. A 30, 2422-2432 (2013)

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