How much CTV coverage is lost when the PTV margin is zero?

Jenghwa Chang, Ph.D [1,2]

1. Radiation Medicine, Donald and Barbara Zucker School of Medicine at Hofstra/Northwell, Lake Success, NY
2. Physics & Astronomy, Hofstra University, Hempstead, NY

INTRODUCTION

Using the clinical target volume (CTV) as the planning target volume (PTV) for brain stereotactic radiosurgery (SRS) and/or brain stereotactic body radiotherapy (SBRT) is a common practice in some centers. Since no treatment process and setup/immobilization apparatus are perfect, the CTV is not fully covered by the prescription dose when the PTV margin is 0. However, the actual CTV coverage lost is not well quantified because proper modeling is still lacking.

AIM

The aim of this study was to develop a statistical model for analyzing the CTV random motion, and provide a quantitative estimate of the CTV coverage loss when the PTV margin is zero.

METHOD

A statistical model previously developed by Chang[1] was used to analyze the CTV setup uncertainties for both translational ($\boldsymbol{S}$) and rotational ($\boldsymbol{R}$) random motions. The combined error $\boldsymbol{E} = \boldsymbol{S} + \boldsymbol{R}$ follows a 3D independent normal distribution with a zero mean and a uniform standard deviation $\sigma_E = \sqrt{\sigma_S^2 + \sigma_R^2}$.

Transforming the probability distribution function (PDF) from the Cartesian to the spherical coordinates, the square of the random motion of CTV in the radial direction is a random variable, noted as P^2, following the chi-square distribution with 3 degree of freedom (DOF):

$$f_{P^2}(u) = \frac{1}{\sqrt{2\pi}}\sqrt{u}e^{-\frac{u}{2}}. \qquad (1)$$

Figure 1 illustrates the motion of CTV relative to the PTV. Both CTV and PTV were assumed to be spherical with the same radius (i.e., zero PTV margin). The part of CTV still covered by the PTV is the shaded area $V_{C\cap P,\rho}$, where ρ is the distance between the centers of the two spheres.

METHOD (CONTINUED)

Percent CTV volume still covered by the PTV after the random motion, is a function of P^2 and therefore also a random variable noted as Q. If $r_P = r_C = r$, $V_{C\cap P,\rho}$ in Figure 1 as a percent of the CTV volume can be solved analytically using the relation of Sphere-Sphere Intersection[2]:

$$\frac{V_{C\cap P,\rho}}{V_C} = \frac{1}{16}\left(2-\frac{\rho}{r}\right)^2\left(4+\frac{\rho}{r}\right). \qquad (1)$$

Let random variable $Q = f\left(\frac{P}{r}\right) = \frac{1}{16}\left(2-\frac{P}{r}\right)^2\left(4+\frac{P}{r}\right)$ or the fraction of CTV still remained in PTV after random motion P, the cumulative distribution function (CDF) of Q is

$$F_Q(q) = Prob(Q<q) = Prob\left(\frac{P}{r} > g^{-1}(q)\right) = Prob\left(P^2 > \left(rg^{-1}(q)\right)^2\right) = \int_{\left(rg^{-1}(q)\right)^2}^{\infty}\frac{1}{\sqrt{2\pi}}\sqrt{u}e^{-\frac{u}{2}}du \qquad (2)$$

The reliability or survival function of Q, can be derived from $F_Q(q)$ as:

$$S_Q(q) = Prob(Q>q) = 1 - F_Q(q) = \int_0^{\left(rg^{-1}(q)\right)^2}\frac{1}{\sqrt{2\pi}}\sqrt{u}e^{-\frac{u}{2}}du\ . \qquad (3)$$

We programmed $F_Q(q)$ in Eq. (2) and $S_Q(q)$ in Eq. (3) in an Microsoft Excel spreadsheet using the "CHISQ.DIST()" function for chi square distribution. The inverse function $g^{-1}(q)$ was calculated using the "Solver" function in Excel for finding optimal solutions for various kind of decision problems.

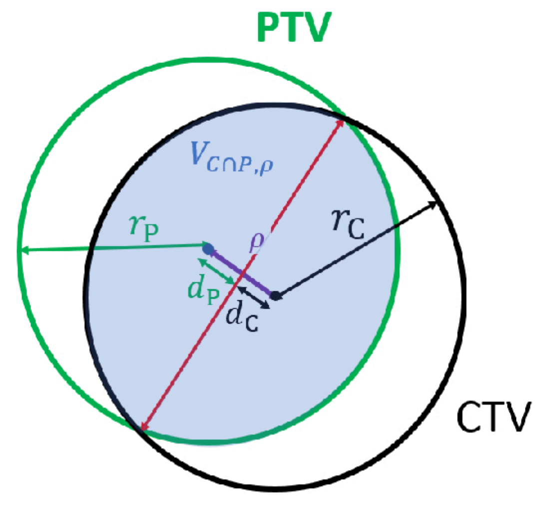

Figure 1. Illustration of the motion of CTV relative to PTV assuming both CTV and PTV are spheres and have the same radius $r_P = r_C = r$. The area shaded in blue, $V_{C\cap P,\rho}$, where ρ is the distance between the centers of the two spheres, denotes the part of CTV still covered by the prescription dose cloud (i.e., PTV).

RESULTS

Figure 2 shows the CDF of random variable Q for percent CTV coverage (by PTV) for various r normalized to σ_E. For $r = 1\sigma_E$, it has a non-zero (~0.26) value indicating that there is a significant possibility that the CTV might be missed completely if the radius of CTV is on the same order as σ_E. For example, assuming $\sigma_E = 0.5\ mm$, there is a ~26% chance that the treatment might miss the target completely if the CTV is ~1mm in diameter. The probability of complete miss decreases quickly with the increasing CTV radius, which is essentially 0 when $r = 2\sigma_E$ or larger, as shown in Figure 2.

Figure 3 illustrates the reliability function $S_Q(q)$, or the probability of Q larger than a given value q. The goal "95% of CTV covered by the prescription dose 95% of the time" is not easily achievable as most curves do not enter the red shaded area except for $r = 50\sigma_E$. Assuming $\sigma_E = 0.5\ mm$, the minimal CTV size that will meet this preference is $r = 42\sigma_E = 21$ mm or 42 mm in diameter, which is usually on the larger side of the targets for single fraction SRS. Common sizes for single fraction SRS are between 5mm and 20 mm, corresponding to the "5", "10" and "20" curves in Figure 3. Within this range, only between 60% ("5" curve) and 90% ("20" curve) of the CTV can be covered by the prescription dose 95% of the time.

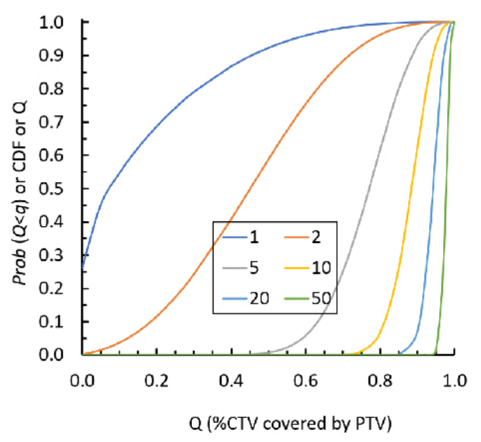

Figure 2. Cumulative distribution function (CDF) of random variable Q for percent CTV coverage (by PTV). It is the probability of Q less than a given value q for various r normalized to σ_E (e.g., "1" means $r = 1\sigma_E$ and "50" means $r = 50\sigma_E$). The probability for complete miss or $Q = 0$ is negligible (i.e., 0) except for $r = 1\sigma_E$ (*Prob*=0.261) and $2\sigma_E$ (*Prob*=0.001).

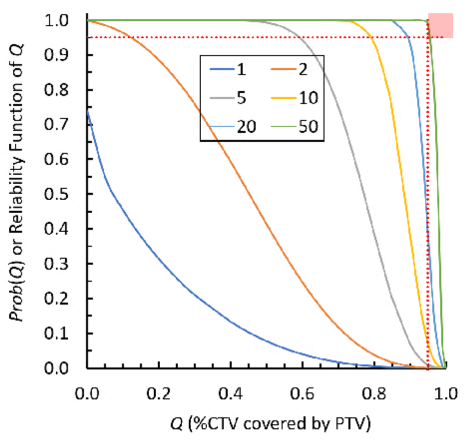

Figure 3. Reliability function of random variable Q for percent CTV coverage (by PTV). It is the probability of Q larger than or at least equal to a given value q for various r normalized to σ_E (e.g., "1" means $r = \sigma_E$ and "50" means $r = 50\sigma_E$). The horizontal and vertical dashed red lines indicate the 95% reliability threshold for respectively q and $S_Q(q)$.

CONCLUSIONS

An analytic solution was derived for modeling the coverage uncertainty when the PTV margin is 0, a common practice in brain SRS/SBRT treatments at some centers. This work provides a quantitative estimate of CTV coverage loss for SRS/SBRT that was traditionally ignored due to the lack of proper modeling and/or the blind belief of the accuracy of immobilization devices and delivery systems. The model predicted that for very small target (i.e., radius on the order of σ_E), there is a significant possibility that the target might be missed completely. In most SRS cases with CTV radius<20 mm, the goal "95% of CTV covered by the prescription dose 95% of the time" is not easily achievable. For the single-isocenter for multiple-targets technique, CTV %coverage probability will decrease with the increased distance between the CTV and the iso-center.

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CONTACT INFORMATION

Jenghwa Chang, Ph.D.
Radiation Medicine, Northwell Health
450 Lakeville Road
Lake Success, NY 11042
Email: jchang24@northwell.edu